whereas the rms fluctuation is given by

$$\frac{\langle[n_e(T) - \langle n_e(T)\rangle]^2\rangle^{1/2}}{\langle n_e(T)\rangle} = \frac{[\overline{(\Delta n_e)^2}]^{1/2}}{\overline{n_e(T)}} = \frac{|n_e\{\bar{T} + [\overline{(\Delta T)^2}]^{1/2}\} - n_e\{\bar{T} - [\overline{(\Delta T)^2}]^{1/2}\}|}{n_e\{\bar{T} + [\overline{(\Delta T)^2}]^{1/2}\} + n_e\{\bar{T} - [\overline{(\Delta T)^2}]^{1/2}\}} \quad (8)$$

in which $[\overline{(\Delta T)^2}]^{1/2} = [\langle(T - \bar{T})^2\rangle]^{1/2}$.

Using the Saha equation and neglecting density fluctuation,[1]

$$\frac{\overline{n_e(T)}}{n_e(\bar{T})} = \frac{1}{2}\left\{(1+\epsilon)^{3/4}\exp\left[\frac{B}{2\bar{T}}\frac{\epsilon}{(1+\epsilon)}\right] + (1-\epsilon)^{3/4}\exp\left[-\frac{B}{2\bar{T}}\frac{\epsilon}{(1-\epsilon)}\right]\right\} \quad (9)$$

$$\frac{[\overline{(\Delta n_e)^2}]^{1/2}}{\overline{n_e(T)}} = \frac{\left|(1+\epsilon)^{3/4}\exp\left[\frac{B}{2\bar{T}}\frac{\epsilon}{(1+\epsilon)}\right] - (1-\epsilon)^{3/4}\exp\left[-\frac{B}{2\bar{T}}\frac{\epsilon}{(1-\epsilon)}\right]\right|}{(1+\epsilon)^{3/4}\exp\left[\frac{B}{2\bar{T}}\frac{\epsilon}{(1+\epsilon)}\right] + (1-\epsilon)^{3/4}\exp\left[-\frac{B}{2\bar{T}}\frac{\epsilon}{(1-\epsilon)}\right]} \quad (10)$$

in which $\epsilon = [\overline{(\Delta T)^2}]^{1/2}/\bar{T}$, and B is the ionization energy. When $\epsilon \ll 1$, Eqs. (9) and (10) take the respective forms

$$\frac{\overline{n_e(T)}}{n_e(\bar{T})} = \cosh\frac{B\epsilon}{2\bar{T}} + 0(\epsilon) \quad (9a)$$

$$\frac{\left[\overline{(\Delta n_e)^2}\right]^{1/2}}{\overline{n_e(T)}} = \tanh\frac{B\epsilon}{2\bar{T}} + 0(\epsilon^2) \quad (10a)$$

It is seen that, to lowest order, the preceding are identical to Eqs. (16) and (19) of Ref. 1.

In summary, the results of Ref. 1 concerning the values of the ratios $\overline{n_e(T)}/n_e(\bar{T})$ and $\left[\overline{(n_e)^2}\right]^{1/2}/\overline{n_e(T)}$ have been shown to be based on a physically unrealistic probability distribution for the temperature, the distribution consisting of two equally weighted delta functions; i.e., a special case of the marble-cake model. It then follows that the analysis of Ref. 1 is equivalent to simply assuming that, for any function $f(T)$, the expected value is the arithmetic average of $f\{\bar{T} + [\overline{(\Delta T)^2}]^{1/2}\}$ and $f\{\bar{T} - [\overline{(\Delta T)^2}]^{1/2}\}$, and that the rms fluctuation is one-half of the magnitude of the difference of $f\{T + [\overline{(\Delta T)^2}]^{1/2}\}$ and $f\{T - [\overline{(\Delta T)^2}]^{1/2}\}$.

References

[1] Demetriades, A., "Electron fluctuations in an equilibrium turbulent plasma," AIAA J. **2**, 1347–1349 (1964).

[2] Friedman, B., *Principles and Techniques of Applied Mathematics* (John Wiley and Sons, New York, 1956), Chaps. 3 and 4.

[3] Herlin, M. A. and Herrmann, J., "Semi-annual technical summary report to ARPA," Sec. II-B, Massachusetts Institute of Technology Lincoln Lab. (March 1963).

[4] Proudian, A. and Feldman, S., "Some theoretical predictions of mass and electron density oscillations based on a simple model for turbulent wake mixing," AIAA Preprint 64-21 (January 1964); also AIAA J. **3**, 602–609 (1965).

Reply by Author to F. Lane and S. L. Zeiberg

Anthony Demetriades*

Philco Corporation, Newport Beach, Calif.

THIS writer is indebted to Lane and Zeiberg for their observation on the probability distribution implied by the results of Ref. 1. In fact, appropriate reservations on the distribution implied by Eq. (12) of the latter reference were also voiced in the same paper, although the form of the distribution itself was not derived.

The applicability of the results of Ref. 1 to the inviscid-mixing or marble-cake model[2, 3] had also occurred to the writer. However, whereas in its early versions the latter model indicated no upper limit in the normalized electron density fluctuations (see Fig. 9 of Ref. 3), the results of Ref. 1 showed a definite upper limit (of unity) of the normalized fluctuations. A tentative explanation of this discrepancy is that the marble-cake formulation is unable to distinguish between the pseudoaverage $n_e(\bar{T})$ and the numerically much larger proper average $\overline{n_e(T)}$. Until the latter formulation is completed to the point where it can be used to compute electron-density fluctuations in a more definitive sense, its identification with the results of Ref. 1 may be misleading.

Further calculations, such as those shown in Ref. 1, have been performed at this laboratory using a Gaussian, a parabolic, and a square probability distribution function. The results are shown in Fig. 1. These indicate that, for all reasonably well-behaved distributions, the fluctuation $[\overline{(\Delta n_e)^2}]^{1/2}/\overline{n_e(T)}$

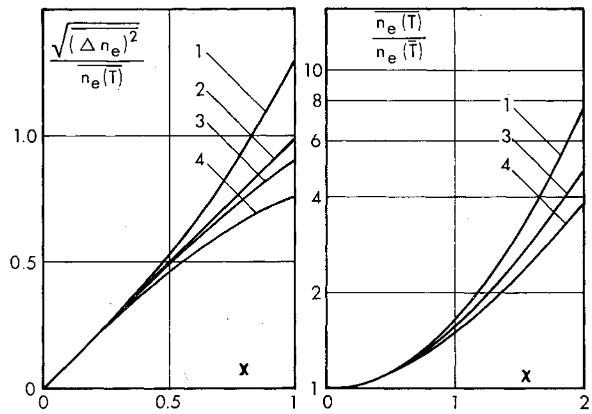

Fig. 1 Dependence of electron density fluctuations (left) and proper average density (right) on virtual temperature fluctuation for various probability distributions: 1) Gaussian, 2) parabolic, 3) square, 4) Ref. 1, Eq. (12).

is proportional to the virtual fluctuation χ for small values of the latter; for air, this corresponds to temperature fluctuations below about 3% for a temperature of 3000°K. This is understandable because for small χ the computation of the higher-order correlations, and, hence, the use of a distribution function, are unnecessary [see Eq. (15), Ref. 1]. However, in contrast to the results of Ref. 1, the previously mentioned distributions do not force the electron fluctuations to any limit as χ increases. On the other hand, all such distribution functions show that the difference between the pseudoaverage and the proper average electron density is even larger than that predicted by Ref. 1.

References

[1] Demetriades, A., "Electron fluctuations in an equilibrium turbulent plasma," AIAA J. **2**, 1347–1349 (1964).

[2] Feldman, S. and Proudian, A., "Some theoretical predictions of *m* mass and electron density oscillations based on a simple model of turbulent wake mixing," AIAA Preprint 64-21 (January 1964); also AIAA J. **3**, 602–609 (1965).

[3] Lin, S. C. and Hayes, J. E., "A quasi-one-dimensional model of chemically reacting turbulent wakes of hypersonic objectis," AVCO Everett Research Lab. RR157 (July 1963).

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* Principal Scientist, Fluid Mechanics Research Department, Aeronutronic Division. Associate Fellow Member AIAA.